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F-term strings in the Bogomolnyi limit are also BPS states

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Abstract: We derive the Bogomol'nyi equations for supersymmetric abelian F-term cosmic strings in four-dimensional flat space and show that, contrary to recent statements in the literature, they are BPS states in the Bogomol'nyi limit, but the partial breaking of supersymmetry is from $N = 2$. The second supersymmetry is not obvious in the $N = 1$ formalism, so we give it explicitly in components and in terms of a different set of $N = 1$ chiral superfields. We also discuss the appearance of a second supersymmetry in D-term models, and the relation to $N = 2$ F-term models. The analysis sheds light on an apparent paradox raised by the recent observation that D-term strings remain BPS when coupled to $N = 1$ supergravity, whereas F-term strings break the supersymmetry completely, even in the Bogomol'nyi limit. Finally, we comment on their semilocal extensions and their relevance to cosmology.

Keywords: Solitons Monopoles and Instantons, Spontaneous Symmetry Breaking, Supersymmetric Effective Theories, Cosmology of Theories beyond the SM.

Contents

1. Introduction

For a number of years now there has been considerable interest in the cosmological impact of topological defects from supersymmetric theories, in particular cosmic strings. Until now most of the efforts had to do with identifying the cause of inflation. $N = 1$ supersymmetric models typically contain complex scalar fields, often with flat directions in their potential, and so are natural candidates for inflationary models. But they are also natural candidates to form topological defects at the end of inflation. Cosmic strings in supersymmetric theories can exhibit interesting new properties with respect to their non-supersymmetric counterparts. In particular, both the fermionic zero modes arising from partial supersymmetry breaking and the bosonic zero modes coming from flat directions or Goldstone bosons can have a very serious impact on the cosmological evolution of such defects.

More recently, interest was further revived by the realization that the soliton solutions in four-dimensional supergravity might be directly identified with ten-dimensional superstring states whose low energy limit they represent [1]–[3]. If these supergravity defects have a potentially measurable effect on the early Universe, comparison with cosmological data might provide direct information about superstring/M theory. One idea that is central to this philosophy is that the BPS states in a supersymmetric model will survive all sorts of deformations of the model. They provide the necessary connection with the higher energy theory.

It is well known that solitons whose mass saturates a Bogomol'nyi-type bound generically lead to partial breaking of supersymmetry, and this in turn protects the energy bound from receiving quantum corrections. States which preserve only a subset of the supersymmetry present in the vacuum are known are BPS states (after Bogomol'nyi-Prasad-

Sommerfield monopoles) and they are global minima of the energy in their topological sector. Although these two conditions are different, they are connected by the appearance of central charges in the supersymmetry algebra [4]–[8]. Bogomol'nyi bounds usually survive the coupling to gravity [9], and we are not aware of any examples where coupling to gravity destroyed the BPS nature of the states.

Dvali, Kallosh and van Proeyen [1] addressed the interesting question of what supersymmetric BPS vortex solutions are possible in $N = 1$ supergravity. In the absence of gravity, this question was considered by Davis, Davis and Trodden [10] some years ago. There are two known ways to spontaneously break a gauge symmetry in $N = 1$ (globally) supersymmetric theories: either by using Fayet-Iliopoulos D-terms (which only works in the abelian case) or by adding F-terms to the superpotential. The Higgs mechanism then leads to the formation of Nielsen-Olesen strings which are usually known as D-term strings and F-term strings respectively. The existence of these string solutions in $N = 1$ supergravity was established by Morris [11], although he did not consider the supersymmetry breaking. In 2+1 dimensions the problem was analysed in [12, 13].

Davis, Davis and Trodden concluded that (global, $N = 1$) supersymmetry was partially broken by D-term strings and fully broken by F-term strings, even in the Bogomol'nyi limit. Dvali, Kallosh and van Proeyen conclude that this result is still true when the models are coupled to $N = 1$ supergravity.

Their results would seem to put into question the BPS nature of the F-strings in the Bogomol'nyi limit. In fact, the issue is more subtle. As one might expect on general grounds [6]–[8] and [14], the globally supersymmetric model has a second supersymmetry in the Bogomol'nyi limit which is not evident in $N = 1$ language. As we will show, before coupling to gravity both D- and F-term strings are BPS states *that preserve 1/2 of the* $N = 2$ *supersymmetry* [37]. It is therefore remarkable that in $N = 1$ supergravity D-term strings remain BPS states, whereas F-term strings break all (local) supersymmetries *even in the Bogomol'nyi limit* [1].

While this result appears paradoxical, we should quickly point out that there is no contradiction, in principle. First of all, the global limit of $N = 2$ local supersymmetry is not at all trivial. Secondly, the Higgs mechanism that gives rise to the strings is due to constant Fayet-Iliopoulos terms, which are known to be incompatible with $N = 2$ supergravity (see [15] for references). Although the Bogomol'nyi bound survives coupling to bosonic gravity, the coupling to $N = 1$ supergravity breaks the SU(2) global symmetry that relates F- and D-term models in the global case and so we should not expect F-term and D-term models to remain equivalent once they are coupled to $N = 1$ supergravity. We will get back to this point at the end of the paper.

In D-term models the second supersymmetry only appears when a specific trilinear coupling is present in the superpotential. The trilinear coupling has to satisfy a Bogomol'nyi-looking relation that we call the superBogomol'nyi limit[1] (because the ordinary Bogomol'nyi limit is always satisfied in D-term models).

[1]The term superBogomol'nyi is sometimes used in the literature to refer to the ordinary Bogomol'nyi limit in F-term models; here we mean something different.

The purpose of this paper is to take over where [10] left off and analyse the effect of the second supersymmetry in both F-term models in the Bogomol'nyi limit and D-term models in the superBogomol'nyi limit. We have made a point of avoiding the $N = 2$ formalism, which is not so familiar to cosmologists.[2] In some sense this is an attempt to translate the $N = 2$ results into $N = 1$ language. Since the second supersymmetry has the opposite chirality, we make it explicit in a (non-standard) way that makes partial breaking of supersymmetry easier to study.

2. Before supersymmetry

We begin by considering the bosonic abelian Higgs model, with action

$$S = \int d^4x \left[|D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} (|\phi|^2 - \eta^2)^2 \right], \tag{2.1}$$

where A_μ is a U(1) gauge field and ϕ is a complex scalar of charge e ,

$$D_\mu \phi = (\partial_\mu + i\, e\, A_\mu)\, \phi\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{2.2}$$

We want to study the properties of straight, static vortices along the z-direction so we drop the t- and z-dependence and set $A_t = A_z = 0$. Their energy per unit length is

$$E = \int d^2x \left[|D_1\phi|^2 + |D_2\phi|^2 + \frac{1}{2} B^2 + \frac{\lambda}{2} (|\phi|^2 - \eta^2)^2 \right], \tag{2.3}$$

where $B = \partial_1 A_2 - \partial_2 A_1$ is (the z-component of) the magnetic field. Finite energy configurations must satisfy $|\phi| \to \eta$ as $r \to \infty$ (the vacuum manifold is a circle) but also $D_\mu \phi \to 0, B \to 0$ faster than $1/r$. If we choose the gauge $A_r = 0$, cylindrically symmetric configurations tend to $\phi(r, \theta) \sim \eta e^{in\theta}$, $A_\theta(r, \theta) \sim -n/(er)$, as $r \to \infty$, and this means that the total magnetic flux in the plane perpendicular to the string is quantized,

$$\int d^2x B = \oint \vec{A} \cdot \vec{dl} = -\frac{2\pi n}{e}\,. \tag{2.4}$$

Following Bogomol'nyi we use the identity $[D_1, D_2]\phi = ieB\phi$ and an integration by parts to rewrite the energy as

$$E = \int d^2x \left[|(D_1 \pm iD_2)\phi|^2 + \frac{1}{2} [B \mp e(|\phi|^2 - \eta^2)]^2 + \frac{\lambda - e^2}{2} (|\phi|^2 - \eta^2)^2 \right] \mp$$
$$\mp \eta^2 e \int d^2x\; B\,. \tag{2.5}$$

We have omitted a boundary term, the curl of a current $\vec{J} = -i\phi^* \vec{D}\phi$ which vanishes at $r = \infty$. The last term is the topological charge $\mp\eta^2 e \int B = \pm 2\pi n \eta^2 = T$. If $\lambda = e^2$, the energy has a lower bound $E \geq |T|$, and the minimum energy configurations ($E = |T|$) are those that satisfy

$$\begin{aligned} &(D_1 + iD_2)\phi = 0\,, \qquad B - e(\phi^2 - \eta^2) = 0 \qquad \text{if} \quad n > 0 \\ &(D_1 - iD_2)\phi = 0\,, \qquad B + e(\phi^2 - \eta^2) = 0 \qquad \text{if} \quad n < 0\,. \end{aligned} \tag{2.6}$$

[2]See [16]–[18] and [37] for a full $N = 2$ treatment

The condition $\lambda = e^2$ is known as the Bogomol'nyi limit. In physical terms, it states that the mass of the scalar fluctuations $m_s = \sqrt{2\lambda}\eta$ is equal to the mass of the vector fluctuations $m_v = \sqrt{2}e\eta$. Perhaps for this reason the Bogomol'nyi limit is sometimes called the supersymmetric limit, although this is somewhat misleading because it is not a necessary condition for $N = 1$ supersymmetrization[3] (see the F-term models below).

Notice that if $\lambda < e^2$ (or $m_s < m_v$) the last term in the integral is not positive, so we cannot deduce a lower bound on the energy. If $\lambda > e^2$ ($m_s > m_v$) we have a lower bound, $E \geq |T|$, but it can never be attained by a vortex configuration since the conditions $B \pm e(\phi^2 - \eta^2) = 0$ and $(\phi^2 - \eta^2) = 0$ taken together imply $B = 0$, which is incompatible with a total magnetic flux of $2\pi n/e$ for non-zero n ($n = 0$ gives $\phi = \pm\eta$, the vacuum).

We can take $n > 0$ without loss of generality. From now on we will restrict ourselves to the lowest non-trivial topological sector, $n = 1$. If we consider cylindrically symmetric configurations,

$$\phi = \eta f(r) e^{i\theta} \qquad A_\theta = -\left(\frac{1}{e}\right)\frac{a(r)}{r} \tag{2.7}$$

with $f(0) = a(0) = 0$, $f(\infty) = a(\infty) = 1$, the equations (2.6) become

$$f' + \frac{a-1}{r} f = 0 \qquad \frac{a'}{r} + e^2\eta^2(f^2 - 1) = 0 \tag{2.8}$$

known as the Nielsen-Olesen equations.

In what follows we will consider two supersymmetrizations of the abelian Higgs model which allow Nielsen-Olesen vortices as stable solutions. The names "F-term" and "D-term" come from $N = 1$ supersymmetry, where they refer to the F- and D- auxiliary fields of the $N = 1$ chiral and gauge superfields, respectively.

Before we go into the supersymmetrizations of the abelian Higgs model, we should mention the semilocal model, which is obtained when the charged field is not a complex scalar but an SU(2) doublet (ϕ_1, ϕ_2) [19]. The presence of Goldstone bosons changes the stability properties of the strings in a dramatic way. Although magnetic flux is still quantized on all solutions with finite energy per unit length, there is no guarantee that the flux will be confined to a core of finite size. String configurations are only stable if $m_s/m_v < 1$. In the Bogomol'nyi limit the string is only neutrally stable and now there is a zero mode which makes the magnetic flux spread to an arbitrarily large area. This zero model is easily excited in a cosmological context, so one does not expect stringy configurations to appear in a cosmological phase transition. It turns out that supersymmetric versions of the semilocal model appear naturally in some brane inflation scenarios and other superstring-inspired models [20]–[22].

3. $N = 1$ supersymmetric abelian Higgs models

In the next three sections we review and expand the analysis of [10] to F-term models in the Bogomol'nyi limit and D-term models in the superBogomol'nyi ($N = 2$) limit. Unless otherwise stated, we follow the conventions of [10]. In particular, from now on $e = g/2$.

[3] Actually, the charged scalar and the gauge field must always belong to different supermultiplets.

JHEP08(2004)050

Let us consider supersymmetric QED, containing an abelian vector superfield V, and m chiral superfields Φ_i, $(i = 1, \dots, m)$ with U(1) charges q_i. In order to have a theory without anomalies which does not leave the gauge symmetry unbroken, we need at least 3 different chiral superfields Φ_0, Φ_+, Φ_- with charges $0, +1, -1$ respectively. The detailed field content of these models is the following: two charged chiral superfields $\Phi_\pm = (\phi_\pm, \psi_\pm, F_\pm)$, one neutral chiral superfield $\Phi_0 = (\phi_0, \psi_0, F)$ and the vector superfield $V = (A_\mu, \lambda, D)$ in the Wess-Zumino gauge. The fields $\phi_\pm$ and ϕ_0 are complex scalar fields; A_μ is a U(1) gauge field; $\psi_\pm$, ψ_0 and λ are Weyl fermions; and $F_\pm$, F and D are auxiliary fields.

We can also have a superpotential, the most general form for a renormalisable theory being

$$W(\Phi_i) = a_i \Phi_i + \frac{1}{2} b_{ij} \Phi_i \Phi_j + \frac{1}{3} \Phi_i \Phi_j \Phi_k \,. \tag{3.1}$$

As each term in the superpotential has to be gauge invariant, $a_i \neq 0$ only if $q_i = 0$, $b_{ij} \neq 0$ only if $q_i + q_j = 0$ and $c_{ijk} \neq 0$ only if $q_i + q_j + q_k = 0$. When the model is abelian (which is our case) we can also add a Fayet-Iliopoulos term of the form $\xi_3 D$ (the reason for the subindex in ξ_3 will become clear later).

The lagrangian density for these models in Wess-Zumino gauge is given by [10]

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - U \tag{3.2}$$

with

$$\begin{aligned}
\mathcal{L}_B &= |D_\mu \phi_i|^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
\mathcal{L}_F &= -i \psi_i \sigma^\mu D_\mu \bar{\psi}_i - i\, \lambda_i \sigma^\mu \partial_\mu \bar{\lambda}_i \\
\mathcal{L}_Y &= i \frac{g}{\sqrt{2}} q_i \phi_i^* \psi_i \lambda - \left(b_{ij} + c_{ijk} \phi_k\right) \psi_i \psi_j + (\text{h.c}) \\
U &= |F_i|^2 + \frac{1}{2} D^2 = |a_i + b_{ij} \phi_j + c_{ijk} \phi_i \phi_j \phi_k|^2 + \frac{1}{2} \left(\xi_3 + \frac{g}{2} q_i \phi_i^* \phi_i \right)^2 ,
\end{aligned} \tag{3.3}$$

where $D_\mu \phi_i = (\partial_\mu + \frac{1}{2} i\, g q_i A_\mu) \phi_i$ and we have substituted the equations of motion of the auxiliary fields into the lagrangian.

4. D-term strings

As discussed in [10], the simplest model containing D-term strings has a lagrangian (3.3), with a Fayet-Iliopoulos term $\xi_3 = -\frac{1}{2} g \eta^2$ and no superpotential. The vacuum manifold in this case is given by

$$U = \frac{g^2}{8} \left(|\phi_+|^2 - |\phi_-|^2 - \eta^2 \right)^2 \,. \tag{4.1}$$

We are interested in a static, straight string with winding number n in the z-direction. The energy per unit length of such an object satisfies a Bogomol'nyi bound:

$$\begin{aligned}
E &= \int \mathrm{d}^2 x \left[|D_1 \phi_+|^2 + |D_2 \phi_+|^2 + |D_1 \phi_-|^2 + |D_2 \phi_-|^2 + \frac{g^2}{8} \left(|\phi_+|^2 - |\phi_-|^2 - \eta^2 \right)^2 + \frac{1}{2} B^2 \right] \\
&= \int \mathrm{d}^2 x \left\{ |(D_1 + i D_2)\, \phi_+|^2 + |(D_1 + i D_2)\, \phi_-|^2 + \frac{1}{2} \left[B - \frac{g}{2} \left(|\phi_+|^2 - |\phi_-|^2 - \eta^2 \right) \right]^2 \right\} + T
\end{aligned} \tag{4.2}$$

where, as in the bosonic case,

$$T = -\frac{g}{2}\eta^2 \int d^2x B = \frac{g}{2}\eta^2 2\pi n \tag{4.3}$$

is the topological charge (in the transverse plane).

We can read off the Bogomol'nyi equations directly from (4.2)

$$\begin{aligned} &(D_1 + iD_2)\phi_+ = 0\,, \qquad (D_1 + iD_2)\phi_- = 0 \\ &B - \frac{g}{2}\left(|\phi_+|^2 - |\phi_-|^2 - \eta^2\right) = 0\,. \end{aligned} \tag{4.4}$$

Even though the vacuum manifold (4.1) includes configurations with $\phi_- \neq 0$, only the $\phi_- = 0$ configurations saturate the Bogomol'nyi bound [17]. The remaining fields $(\phi_+,\ A_\mu)$ take the usual Nielsen-Olesen vortex form (2.7), (2.8).

The condition $\phi_- = 0$ is a direct consequence of the first two Bogomol'nyi (4.4) equations, which imply $(\partial_1 + i\partial_2)(\phi_+\phi_-) = 0$ or, using $z = x + iy$, $\partial_z(\phi_+\phi_-) = 0$. Since $(\phi_+\phi_-)$ is analytic and bounded for all z, it is constant. For a string, with $\phi_+(0) = 0$, the constant must be zero, and so we conclude that $\phi_- \equiv 0$ [17, 23, 24].

Note that we did not have to impose any condition on the parameters of the model, the Bogomol'nyi condition $\lambda = g^2/4$ in (2.1) is automatically satisfied. The string solution saturates the Bogomol'nyi bound, and so we expect partial breaking of supersymmetry. This is easily verified by looking at the supersymmetry transformations of the fermions in the background given by the string (4.4) with all fermions put to zero [10] (the bosonic fields are obviously invariant since their transformations are proportional to the fermions)

$$\begin{aligned} \delta(\psi_+)_\alpha &= i\sqrt{2}\left(\sigma^1 D_1\phi_+ + \sigma^2 D_2\phi_+\right)_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(1)} = 2iD_1\phi_+ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(1)} \\ \delta(\psi_-)_\alpha &= 0 \\ \delta(\psi_0)_\alpha &= 0 \\ \delta(\lambda)_\alpha &= i\left(\sigma^3 B + \frac{g}{2}(|\phi_+|^2 - \eta^2)\right)_\alpha^{\ \beta}\epsilon_\beta^{(1)} = 2iB\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_\alpha^{\ \beta}\epsilon_\beta^{(1)}\,, \end{aligned} \tag{4.5}$$

where $\epsilon^{(1)}$ is the SUSY parameter.

The string solution is invariant under half of the supersymmetry transformations: those with $\epsilon_1^{(1)} = 0$. The other half are broken, they create fermion zero modes. The number of fermionic zero modes is half what one would expect in an arbitrary configuration with positive energy. We say that the string is a $\frac{1}{2}$-BPS state, or that the supersymmetry is half-broken.

As mentioned in the introduction, at a given limit the theory becomes $N = 2$ supersymmetric. We wish to find the second supersymmetry from an $N = 1$ point of view: a reordering of the fields in new multiplets will allow us to show the second supersymmetry explicitly. First, for reasons that will become clear later, we have to add a superpotential to the model:

$$W = \beta\Phi_0\Phi_+\Phi_- \tag{4.6}$$

which modifies the scalar potential to

$$U = \frac{g^2}{8}\left(|\phi_+|^2 - |\phi_-|^2 - \eta^2\right)^2 + \beta^2|\phi_0|^2\left(|\phi_-|^2 + |\phi_+|^2\right) + \beta^2|\phi_+\phi_-|^2\,. \tag{4.7}$$

The new vacuum manifold includes the condition $\phi_- = \phi_0 = 0$ automatically, without any further vacuum selection effect by the strings.

The Yukawa terms after the addition of the superpotential (4.6) are the following:

$$\mathcal{L}_Y = i\frac{g}{\sqrt{2}}\left(\phi_+^*\psi_+\lambda + \phi_-^*\psi_-\lambda\right) - \beta\left(\phi_0\psi_+\psi_- + \phi_+\psi_-\psi_0 + \phi_-\psi_+\psi_0\right) + (\text{h.c.})\,. \tag{4.8}$$

The new supersymmetry will re-shuffle the Yukawa terms. Therefore, in order to leave the Yukawa terms invariant with respect to the new supersymmetry, a specific value for β is needed [13]. We call this limit the superBogomol'nyi limit

$$\beta = \frac{g}{\sqrt{2}} \tag{4.9}$$

Given the D-term model (3.3), with a Fayet-Iliopoulos term $\xi_3 = -\frac{1}{2}g\eta^2$, superpotential (4.6) and in the superBogomol'nyi limit, we can try to find the second supersymmetry from an $N = 1$ point of view:

The second supersymmetry is anti-chiral, and connects the positively (negatively) charged scalar to the charge conjugate of the negatively (positive) charged fermion, and the neutral scalar to the gaugino. We can write it as chiral provided we exchange the scalar particles and their charge conjugates in the multiplets.

Consider the following multiplets:

$$\begin{aligned}
\hat{\Phi}_+ &= \left(\phi_-^*, \psi_+, \hat{F}_+\right) \\
\hat{\Phi}_- &= \left(-\phi_+^*, \psi_-, \hat{F}_-\right) \\
\hat{\Phi}_0 &= \left(\phi_0, i\lambda, \hat{F}_0\right) \\
\hat{V} &= \left(A_\mu, i\psi_0, \hat{D}\right).
\end{aligned} \tag{4.10}$$

This way of reordering the fields into new multiplets is already foreseeing that the neutral chiral multiplet (Φ_0) and the gauge (V) will be part of the same $N = 2$ vector multiplet; and similarly with Φ_+ and Φ_- being part of the same hypermultiplet.

It is easy to check that from a field content given by the "new" $N = 1$ supersymmetric multiplets (4.10), we can obtain the D-term model provided that we add a superpotential of the form $W = \beta\hat{\Phi}_0\hat{\Phi}_+\hat{\Phi}_-$ and a Fayet-Iliopoulos term $\hat{\xi}_3 = \frac{1}{2}g\eta^2 = -\xi_3$ (note the change in sign in the Fayet-Iliopoulos term). The Yukawa terms are identical to (4.8) if we are in the superBogomol'nyi limit. Therefore, we have been able to arrive at the same model from two different sets of supersymmetric multiplets, i.e., the model possesses two supersymmetries.

JHEP08(2004)050

Under this new supersymmetry, we have new fermionic zero modes

$$
\begin{aligned}
\delta(\psi_+)_\alpha &= 0 \\
\delta(\psi_-)_\alpha &= -i\sqrt{2}\left(\sigma^1 D_1\phi_+^* + \sigma^2 D_2\phi_+^*\right)_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(2)} = -2iD_1\phi_+^*\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(2)} \\
\delta\psi_0 &= \left(\sigma^3 B - \frac{g}{2}(|\phi_+|^2-\eta^2|)\right)_\alpha^{\ \beta}\epsilon_\beta^{(2)} = 2B\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}_\alpha^{\ \beta}\epsilon_\beta^{(2)} \\
\delta\lambda &= 0
\end{aligned} \tag{4.11}
$$

Therefore, we have 2 supersymmetries half broken, and a total of four fermionic zero modes. The projected supersymmetry generators $\sigma_+\epsilon^{(1)}, \sigma_-\epsilon^{(2)}$ are the generators of the fermionic zero modes, whereas $\sigma_-\epsilon^{(1)}, \sigma_+\epsilon^{(2)}$ generate the unbroken supersymmetries. The projectors $\sigma_\pm$ are given by:

$$
\sigma_+ = \frac{1}{2}\left(1 - i\sigma^1\sigma^2\right) = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \qquad \sigma_- = \frac{1}{2}\left(1 + i\sigma^1\sigma^2\right) = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}. \tag{4.12}
$$

The D-term model can be made semilocal by the addition of a second pair of charged superfields Ψ_+, Ψ_-. The superpotential

$$
W = \beta\Phi_0(\Phi_+\Phi_- + \Psi_+\Psi_-) \tag{4.13}
$$

gives a scalar potential (note that $\psi_\pm$ are now scalars)

$$
\begin{aligned}
U = {} & \frac{g^2}{8}\left(|\phi_+|^2 + |\psi_+|^2 - |\phi_-|^2 - |\psi_-|^2 - \eta^2\right)^2 + \beta^2|\phi_0|^2\left(|\phi_-|^2 + |\phi_+|^2 + |\psi_-|^2 + |\psi_+|^2\right) + \\
& + \beta^2|\phi_+\phi_- + \psi_+\psi_-|^2 .
\end{aligned} \tag{4.14}
$$

The Bogomol'nyi limit is automatically satisfied for any β and the configurations that saturate the bound have $\phi_- = \phi_0 = \psi_- = 0$. However there is no unique string solution, but a one-parameter family of string-like configurations where the magnetic flux is spread over an arbitrarily large area. The general solution, modulo SU(2) rotations between ϕ_+ and ψ_+,[4] can be given — for suitably chosen $f(r)$, $a(r)$ depending on q_0 — by [25]

$$
\phi_+ = \eta f(r)e^{i\theta} \qquad \psi_+ = q_0\frac{f(r)}{r} \qquad A_\theta = -\frac{2}{g}\frac{a(r)}{r}. \tag{4.15}
$$

All these configurations saturate the energy bound. The parameter q_0 fixes the size for the vortex: for $q_0 = 0$, we recover the Nielsen-Olesen profile (2.7), and for bigger q_0 the width of the resulting string is larger. In fact, a small perturbation on any of the strings of the family will cause the string to become wider [26]. The magnetic flux remains quantized, although in the wider strings it is more spread out (see also [38]).

Irrespective of the value of q_0, the configurations are all BPS. In the superBogomol'nyi limit the second supersymmetry appears and the number of fermionic zero modes doubles [27].

[4]Note that this SU(2) rotation is not related to the $N = 2$ SU(2)

5. F-term strings

F-term string solutions can be obtained with the following superpotential

$$W = \beta\Phi_0 \left(\Phi_+\Phi_- - \frac{\eta^2}{2}\right), \tag{5.1}$$

where the last term is known as an F-term. There is no need of including a Fayet-Iliopoulos term. The potential energy for the present case reads:

$$U = \beta^2 \left|\phi_+\phi_- - \frac{\eta^2}{2}\right|^2 + \beta^2|\phi_0|^2\left(|\phi_+|^2 + |\phi_-|^2\right) + \frac{g^2}{8}\left(|\phi_+|^2 - |\phi_-|^2\right)^2 . \tag{5.2}$$

If $\beta = g^2/2$, we can write the energy per unit length of the system in the Bogomol'nyi way, for a static, straight string with winding number $n = 1$ in the z-direction:

$$\begin{aligned} E = &\int \mathrm{d}^2x \frac{1}{2}\Big[\left|(D_1 + iD_2)(\phi_+ + \phi_-^*)\right|^2 + \frac{1}{2}\left|(D_1 + iD_2)(\phi_- - \phi_+^*)\right|^2 + \\ &\quad + \frac{1}{2}\left[B - \frac{g}{2}(\phi_+\phi_- + \phi_+^*\phi_-^* - \eta^2)\right]^2 + \frac{g^2}{8}\left(i\phi_+\phi_- - i\phi_+^*\phi_-^*\right)^2 + \\ &\quad + \frac{g^2}{8}\left(|\phi_+|^2 - |\phi_-|^2\right)^2 + \frac{g^2}{2}|\phi_0|^2\left(|\phi_+|^2 + |\phi_-|^2\right)\Big] + \\ &+ \frac{g}{2}\eta^2 \int \mathrm{d}^2x B . \end{aligned} \tag{5.3}$$

Note that, contrary to the D-term case, we had to impose the condition (4.9) in order to be able to write the energy in the form (5.3). This is the (ordinary) Bogomol'nyi limit.

The Bogomol'nyi equations can then be immediately read off

$$\begin{aligned} (D_1 + iD_2)(\phi_+ + \phi_-^*) &= 0\,; \qquad (D_1 + iD_2)(\phi_- - \phi_+^*) = 0\,; \\ |\phi_+|^2 - |\phi_-|^2 &= 0\,; \qquad \phi_+\phi_- = \phi_+^*\phi_-^* \\ B - \frac{g}{2}(\phi_+\phi_- + \phi_+^*\phi_-^* - \eta^2) &= 0\,. \end{aligned} \tag{5.4}$$

which can be rewritten as

$$\begin{aligned} \phi_+ &= \phi_-^* \\ (D_1 + iD_2)\phi_+ &= 0 \\ B - g\left(|\phi_+|^2 - \frac{\eta^2}{2}\right) &= 0 \end{aligned} \tag{5.5}$$

Bogomol'nyi equations have been derived before in the literature [28, 29] using the ansatz $\phi_+ = \phi_-{}^*$. We just showed here that the condition $\phi_+ = \phi_-{}^*$ is not an ansatz, but one of the Bogomol'nyi equations.

From the point of view of $N = 1$ supersymmetry, the straight infinite string configuration breaks supersymmetry completely. This can be seen from the fermionic supersymmetry transformations:

$$\begin{aligned}
\delta(\psi_+)_\alpha &= i\sqrt{2}\left(\sigma^1 D_1\phi_+ + \sigma^2 D_2\phi_+\right)_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(1)} \\
\delta(\psi_-)_\alpha &= i\sqrt{2}\left(\sigma^1 D_1\phi_- + \sigma^2 D_2\phi_-\right)_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(1)} \\
\delta(\psi_0)_\alpha &= -g\left(|\phi_+|^2 - \frac{\eta^2}{2}\right)_\alpha^\beta \epsilon_\beta^{(1)} \\
\delta(\lambda)_\alpha &= iB\left(\sigma^3\right)_\alpha^\beta \epsilon_\beta^{(1)}\,.
\end{aligned} \tag{5.6}$$

In the Bogomol'nyi limit, one would expect to have some unbroken supersymmetry remaining, i.e., some of the fermionic transformations should be zero. Applying the Bogomol'nyi equations to those fermionic zero modes we get

$$\begin{aligned}
\delta(\psi_+)_\alpha &= i2\sqrt{2}D_1\phi_+\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(1)} \\
\delta(\psi_-)_\alpha &= i2\sqrt{2}D_1\phi_-\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(1)} \\
\delta(\psi_0)_\alpha &= -B\epsilon_\alpha^{(1)} \\
\delta(\lambda)_\alpha &= iB\left(\sigma^3\right)_\alpha^\beta \epsilon_\beta^{(1)}\,.
\end{aligned} \tag{5.7}$$

Unlike in the D-term case, there is no way of finding a projection of the SUSY parameter that makes all four fermions zero. In the Bogomol'nyi limit some of the entries cancel but the cancellations happen in such a way that there is some fermionic content left. So it appears that there are twice as many fermion zero modes in F-strings than in D-strings.

But in the Bogomol'nyi limit the system has $N = 2$ supersymmetry, and it is closely related to the D-term model (in fact, they are equivalent). So let us try to search for the second supersymmetry as in the previous case: analysis of the Yukawa couplings (similar to the D-term case studied in the previous section) shows that with no Fayet-Iliopoulos term ($\hat\xi_3 = 0$) and superpotential

$$W = \frac{g}{2}\hat\Phi_0\left(\hat\Phi_+\hat\Phi_- + \frac{\eta^2}{2}\right) \tag{5.8}$$

we obtain the lagrangian for F-term strings. The hatted multiplets are the same as before, eq. (4.10). Note the change of sign in the η^2 term in the superpotential. We can then read the new supersymmetric transformations (in the Bogomol'nyi limit)

$$\begin{aligned}
\delta(\psi_+)_\alpha &= i2\sqrt{2}D_1\phi_+\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(2)} \\
\delta(\psi_-)_\alpha &= -i2\sqrt{2}D_1\phi_-\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha(2)} \\
\delta(\psi_0)_\alpha &= B\left(\sigma^3\right)_\alpha^\beta \epsilon_\beta^{(2)} \\
\delta(\lambda)_\alpha &= -iB\epsilon_\beta^{(2)}\,.
\end{aligned} \tag{5.9}$$

Once again, there is no unbroken supersymmetry, until we consider combinations of these two supersymmetries:

$$\left(\delta^{(1)} - \delta^{(2)}\right)\psi_+ = 0 \qquad \left(\delta^{(1)} + \delta^{(2)}\right)\psi_- = 0\,. \tag{5.10}$$

It can easily be seen that

$$\sigma_-\left(\delta^{(1)} + \delta^{(2)}\right) f = 0 \qquad \sigma_+\left(\delta^{(1)} - \delta^{(2)}\right) f = 0 \tag{5.11}$$

for f any fermion in the theory. Thus, those two combinations (with projectors given by (4.12)) leave the supersymmetry unbroken. This shows that the F-term string in the Bogomol'nyi bound is $\frac{1}{2}$-BPS saturated.

The semilocal extension of the F-term model is obvious with a second pair of charged superfields Ψ_+, Ψ_- and superpotential

$$W = \beta\Phi_0\left(\Phi_+\Phi_- + \Psi_+\Psi_- - \frac{\eta^2}{2}\right). \tag{5.12}$$

However in this case the scalar potential is (note that $\psi_\pm$ are again scalars)

$$\begin{aligned} U = \beta^2\left|\phi_+\phi_- + \psi_+\psi_- - \frac{\eta^2}{2}\right|^2 + \beta^2|\phi_0|^2\left(|\phi_+|^2 + |\phi_-|^2 + |\psi_+|^2 + |\psi_-|^2\right) + \\ + \frac{g^2}{8}\left(|\phi_+|^2 + |\psi_+|^2 - |\phi_-|^2 - |\psi_-|^2\right)^2 \end{aligned} \tag{5.13}$$

and it can be away from the Bogomol'nyi limit, so the stability of the string solutions depends on the values of the couplings. If $\beta < g^2/2$ the Nielsen-Olesen strings are stable but if $\beta > g^2/2$ no strings will form (see also [38]).

In the Bogomol'nyi limit the configurations that saturate the bound have $\phi_0 = 0,\ \phi_+ = \phi_-^*,\ \psi_+ = \psi_-^*$. Again there is no unique string solution, but a one-parameter family of string-like configurations where the magnetic flux is spread over an arbitrarily large area. They can be obtained from the D-term ones by an SU(2) rotation described in the next section.

6. The equivalence between D-term and F-term models: P-term models

We have given arguments suggesting that the F-term model in the Bogomol'nyi limit and the D-term model in the super-Bogomol'nyi limit are both $N = 2$ and admit string solutions that break half of the supersymmetries. Both statements can be proved easily in the $N = 2$ formalism. In fact, from the $N = 2$ point of view, the two models are equivalent [1] and we will now illustrate this equivalence (once again, full justification relies on the $N = 2$ analysis).

As anticipated, in $N = 2$ language the neutral scalar field and the gauge field belong to the same ("vector") multiplet. The auxiliary field of the vector multiplet is an SU(2) triplet $\vec{P}$ called a P-term in this case, and from the $N = 1$ point of view it reduces to F- or D-terms for different orientations. Therefore, the SU(2) symmetry existing among the

$N=2$ supercharges rotates F-terms into D-terms. Charged superfields also combine in a non-chiral hypermultiplet of charge $q=+1$, $h=(h_1,h_2)=(\phi_+,\phi_-^*)$ (h_i where $i=1,2$ are in the fundamental representation of SU(2)).

One can add Fayet-Iliopoulos terms of the form $\vec{\xi}\cdot\vec{P}$. The resulting lagrangian, after elimination of the auxiliary fields, has the following potential term for the scalars:

$$U=\frac{g^2}{8}\sum_{i=1}^{3}(h^\dagger\tau^i h-\xi^i)^2 \tag{6.1}$$

that is,

$$\begin{aligned}U=&\frac{g^2}{8}\left[(\phi_+^*,\phi_-)\tau^1\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}-\xi^1\right]^2+\\ &+\frac{g^2}{8}\left[(\phi_+^*,\phi_-)\tau^2\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}-\xi^2\right]^2+\frac{g^2}{8}\left[(\phi_+^*,\phi_-)\tau^3\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}-\xi^3\right]^2,\end{aligned} \tag{6.2}$$

where τ^i are the Pauli matrices. The D-term model is obviously obtained if $\xi_1=\xi_2=0$. To see the relation to F-terms, note that

$$\begin{aligned}\left|\phi_+\phi_--\frac{\eta^2}{2}\right|^2&=\left[\mathrm{Re}\left(\phi_+\phi_--\frac{\eta^2}{2}\right)\right]^2+[\mathrm{Im}(\phi_+\phi_-)]^2\\ &=\frac{1}{4}\left[(\phi_+^*,\phi_-)\tau^1\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}-\eta^2\right]^2+\frac{1}{4}\left[(\phi_+^*,\phi_-)\tau^2\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}\right]^2\end{aligned} \tag{6.3}$$

so it corresponds to $\xi_2=\xi_3=0$.

An SU(2) rotation $h_i\to U_i^j h_j$

$$\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}\to\frac{1}{\sqrt{2}}\begin{pmatrix}\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}n^3 & i\sin\frac{\theta}{2}(n^1-in^2)\\ i\sin\frac{\theta}{2}(n^1+in^2) & \cos\frac{\theta}{2}-i\sin\frac{\theta}{2}n^3\end{pmatrix}\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix} \tag{6.4}$$

will induce an (inverse) SO(3) rotation on the ξ^i of angle $-\theta$ about the $\vec{n}$ axis ($\vec{n}\cdot\vec{n}=1$). For instance, to go from D-term $\vec{\xi}=(0,0,\eta^2)$ to F-term $\vec{\xi}=(\eta^2,0,0)$ we need a rotation of $-\pi/2$ about the y-axis, which is obtained by an SU(2) transformation[5] of the scalars with $\theta=\pi/4$, $\vec{n}=(0,1,0)$:

$$\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix}\to\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}\phi_+\\ \phi_-^*\end{pmatrix} \tag{6.5}$$

in agreement with what we found in section 5.

7. Discussion

Let us summarise here the main points illustrated in this paper:

• D-term models are always in the Bogomol'nyi limit, but can be $N=1$ or $N=2$ depending on the superpotential. In all cases the Nielsen-Olesen string breaks half of the supersymmetries, so in the superBogomol'nyi limit the number of fermion zero modes is twice that outside the Bogomol'nyi limit. Outside the superBogomol'nyi limit there are two chiral zero modes, and a second pair of antichiral zero modes appear in the superBogomol'nyi limit.

[5]This transformation differs from the analogous one in [18] by a factor of $i\sigma^2$

- F-term strings are $N = 1$ supersymmetric even away from the Bogomol'nyi limit, and become $N = 2$ in the Bogomol'nyi limit. Supersymmetry breaking is complete in the $N = 1$ case and there is partial breaking in the $N = 2$ case. As a result, the number of fermion zero modes is always four, two of each chirality.

- The F-term model in the Bogomol'nyi limit is equivalent to the D-term model in the superBogomol'nyi limit. This is obvious in the manifestly $N = 2$ supersymmetric formulation, where the two are related by an SU(2) rotation of the supercharges.

- D-term models with zero superpotential have a flat direction $|\phi_+|^2 - |\phi_-|^2 = \eta^2$. The Bogomol'nyi equations force $\phi_- = 0$ for the BPS string (a "vacuum selection" effect). Addition of a superpotential $W = \beta\Phi_0\Phi_+\Phi_-$ lifts the flat direction but does not change the structure of the D-term strings. For a specific value of β a second supersymmetry appears of the opposite chirality.

- Both types of models can be made semilocal by the addition of a second pair (or more) of charged superfields with identical couplings. The stability of strings in the semilocal model depends on the ratio of the scalar and gauge masses, so F-term models can only have stable strings if $\lambda < e^2$ (the same condition as for type-I superconductors) or neutrally stable strings in the Bogomol'nyi limit. D-term models are always in the Bogomol'nyi limit so they lead to neutrally stable strings.

This raises a number of interesting open questions:

- In D-term models the semilocal zero mode survives coupling to gravity, including $N = 1$ supergravity, and this is important for the viability of D-term inflation, and in particular of some brane inflation models [20]–[22] and [30]–[32]. But it also shows that the Nielsen-Olesen D-term strings are not the only string-like BPS states in $N = 1$ supergravity, and the task of identifying D-strings in the low energy limit may not be as straightforward as had been hoped.

- In the F-term case the situation is unclear, and the fate of the zero mode has not been analysed so far. We expect the answer to depend on the details of the supersymmetry breaking [24, 38].

Let us get back to the paradox mentioned in the introduction. If one looks at the bosonic sector alone, both F-term strings and D-term strings satisfy Bogomol'nyi bounds which are known to survive coupling to (bosonic) gravity [9]. In fact we have shown that before coupling to gravity, the D-term and F-term models considered in [1] are $N = 2$ supersymmetric and their D-term and F-term strings are completely equivalent. They both carry a conserved topological charge, which appears as a central charge (current) in the supersymmetry algebra, and the naive expectation is that they should both survive as BPS states in the presence of gravity. And yet, it was shown in [1] that in the $N = 1$ supergravity model only the D-term strings remain BPS; F-term strings break all the supersymmetries *even in the Bogomol'nyi limit.*

The problem is that coupling to $N = 1$ supergravity involves making local *one* of the two supersymmetries present in the Bogomol'nyi limit, and we have seen that the broken and unbroken supersymmetries are specific combinations. If we make local one of the combinations $(Q_1 \pm Q_2)/\sqrt{2}$, the resulting $N = 1$ strings will appear to be D-term strings and exhibit partial breaking of (now local) supersymmetry as described in [1]. Any other choice, for instance making Q_1 local, gives rise to a complete breaking of the local supersymmetry by the strings. This is the choice that was implicit in [1]. In terms of the central charges in the supersymmetry algebra, the difference is that in the F-term model the central charge appears in the mixed commutators of the two supersymmetries, so BPS states are only possible in the $N = 2$ case, while in the D-term model the central charge appears in each of the two $N = 1$ subalgebras, so partial breaking of supersymmetry is possible in both the $N = 1$ and $N = 2$ cases [29].

We would like to end with a comment about the cosmological relevance of BPS solutions and partial breaking of supersymmetry. Dvali, Kallosh and van Proeyen were interested in identifying how certain superstring and brane configurations look like from the low energy (i.e. supergravity) point of view. However, there is another reason why partially broken supersymmetry is important for cosmology: it can give rise to chiral vortons, whose cosmological implications can be quite serious as they can disrupt nucleosynthesis or even overclose the mass of the Universe unless they are formed at a relatively low energy [33]–[36]. This was the original motivation behind the study by Davis, Davis and Trodden, and it is still valid. The presence of chiral vortons imposes serious constraints on any particle physics model, so in principle it could also help to constrain superstring/M theory compactifications.

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